

VIBRATION AND BUCKLING STUDIES OF  
PRETENSIONED STRUCTURES

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Large Space Systems Technology

1981

Third Annual Technical Review

November 16-19, 1981

## INTRODUCTION

Many proposed large space structures make use of the low mass and deployability of pretensioned structures to achieve efficient designs. These structures use tension elements (cables, rods and membranes) to provide stiffness and stability to structural systems. To understand the fundamental structural characteristics of pretensioned structures, analyses and tests of some simple configurations have been performed. The buckling and vibration behavior of a pretensioned stayed column have been studied in detail.

Fig. 1 shows several areas which have been identified as needs in pretensioned structure research. The first four will be discussed with the aid of data obtained during analyses and tests of the stayed column. Deployment will not be discussed.

- 0 ANALYTICAL MODELING AND VERIFICATION
- 0 EFFECTS OF FABRICATION IMPERFECTIONS
- 0 OPTIMUM PRESTRESS LEVEL
- 0 EFFECTS AND ANALYSIS OF NONLINEARITIES
- 0 CONTROLLED SEQUENTIAL DEPLOYMENT

Figure 1

## Pretensioned Stayed Column

The test article, shown in Fig. 2, consists of 24 cables or stays attached to a central tube. The column length is 5.2 m and the mass, including joints and fittings, is 0.504 kg. Three (3) stay planes consisting of eight (8) stays and one radial spoke are used to provide isotropic stiffness. Lateral stiffness provided to the central tube by the stays was designed to insure Euler buckling in individual bays. Imperfection analysis was used to determine the prestress level required to prevent stay slackening due to static compressive loading.

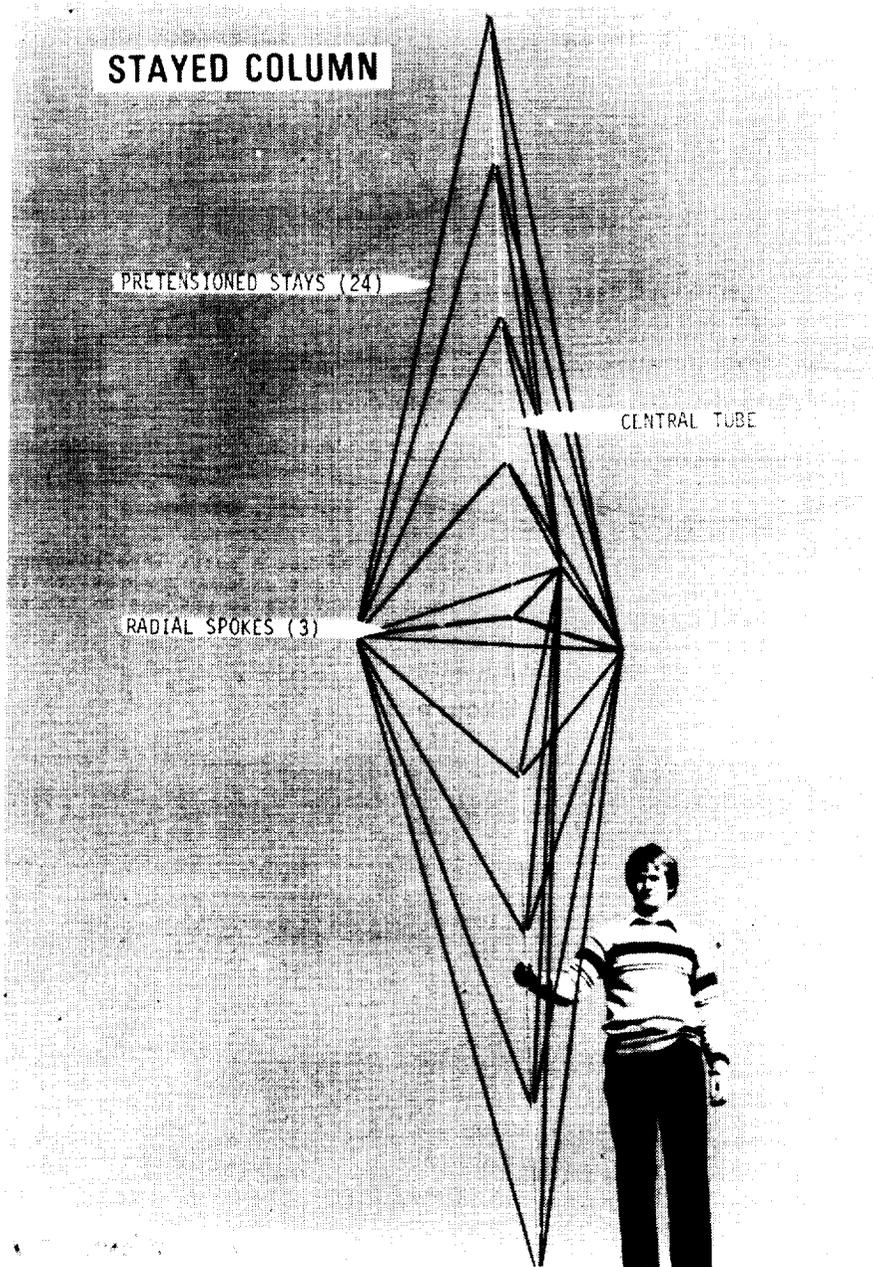


Figure 2

## Constant-Force Spring Capsules Used to Reduce Imperfections

To provide axial stiffness, the stays must be in tension at all times. The pretension force for each set of stays is provided by a constant force spring capsule as shown in Figure 3. This capsule uses the Euler load of three buckled stainless steel strips to maintain the sum of the axial components of tension in the three stays to be constant. If the central tube bending stiffness is negligible, the spring capsule will absorb errors in stay length and distribute an equal force to each stay. However, the central tube bending stiffness is not negligible in most cases and prevents the force from being evenly distributed. Nevertheless, the use of a spring capsule to pretension the stays reduces the effects of fabrication imperfections by allowing the stays to move along the central tube axis. Rigidly attaching the stays to the central tube would preclude axial motion and result in larger fabrication imperfections.

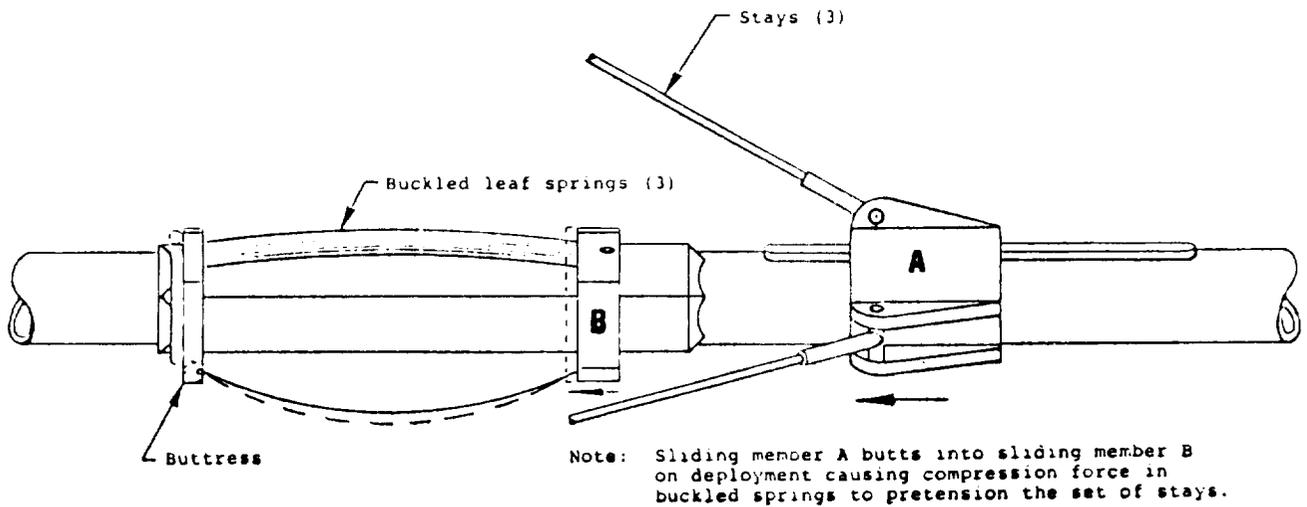


Figure 3

## Buckling and Vibration Test Fixture

All buckling and vibration tests of the stayed column were performed in the test fixture shown in Fig. 4. Non-contacting proximity probes were used to measure static and dynamic deflections. Static axial load was applied to the column by adding weight to the load pan. Only the lateral vibrational characteristics of the column were analyzed. Consequently, the electrodynamic shaker was always oriented perpendicular to the column axis. Vibration tests were performed with and without axial load.

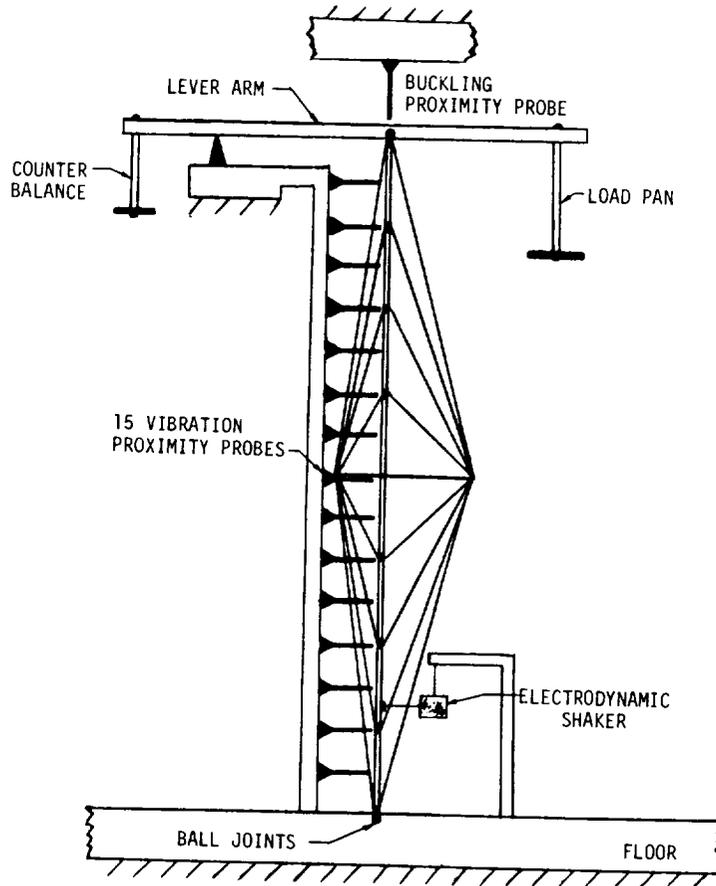


Figure 4

## Buckling Behavior and Analyses

As mentioned earlier, the column was designed to produce Euler buckling between bays. The mode shape, shown in Figure 5, was computed by finite element analysis and shows indeed that the design results in such a mode. The initial precompression is highest in the center bays and can be noticed by larger deflections in these bays. Two finite element analyses were used to analyze the buckling load: Model 1 used linear stiffness terms and model 2 used "exact" transcendental stiffness terms. Both analyses agreed within 2 percent of the observed buckling load of 277.4 N. The use of spring capsules permits a highly determinate load path with taut stays and permits accurate calculations of buckling loads.

### ANALYTICAL AND EXPERIMENTAL BIFURCATION BUCKLING LOAD

	BUCKLING LOAD, N
MODEL 1	280.67
MODEL 2	277.22
EXPERIMENTAL	277.4

### MODE SHAPE

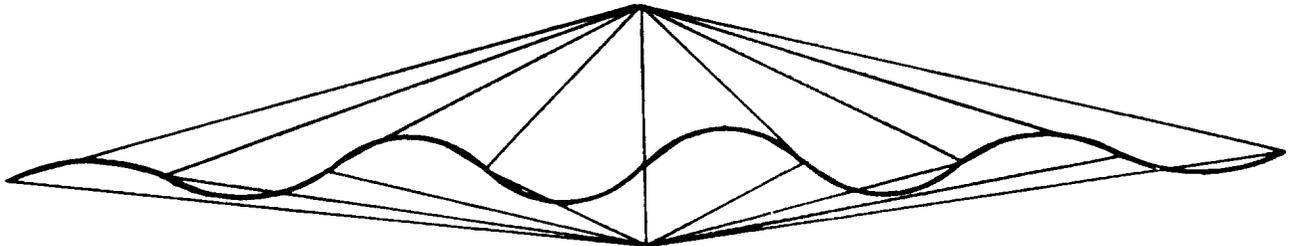


Figure 5

## Unusual Post Buckling Behavior Observed

The end shortening that occurs after buckling results in slackening of the stays. When the stays become slack, they no longer provide the lateral stiffness required to form nodes in the central tube. Stay slackening results in the central tube buckling in an overall bending mode. Since the buckling load of the slack stay mode is much less than the load associated with the taut stay mode, the column exhibits a post buckling restoring force of approximately 1/64 the original buckling load. Figure 6 shows a graphical representation of the stayed column's load deflection path in the post buckling region. To return to the original shape the column compressive load must be reduced below the slack stay buckling load. This behavior will require the designer to insure a safe margin between the design operating load and the buckling load.

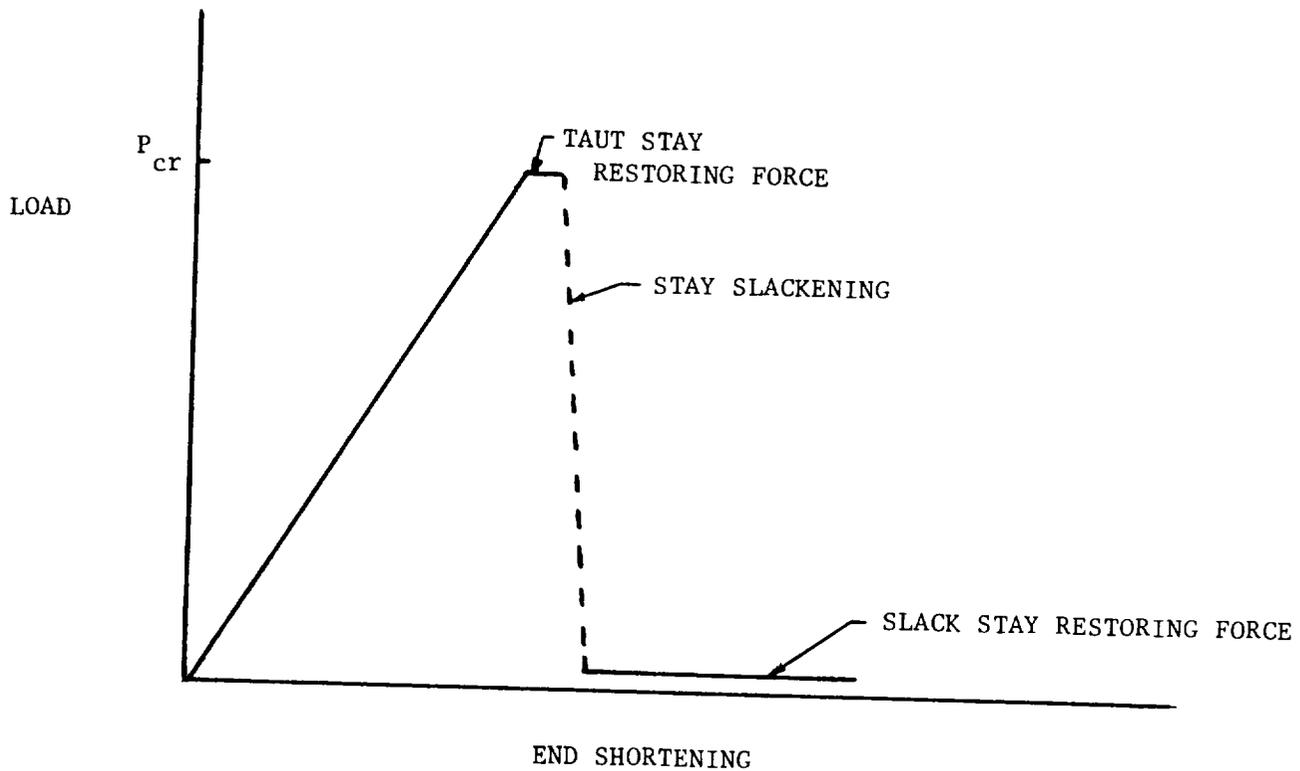


Figure 6

## Frequency Spectrum

Response of the column to dynamic excitation is difficult to analyze due to the large number of natural frequencies or high modal density of the structure. Figure 7 shows the quadrature component of displacement at the 15 proximity probe locations indicated in Figure 4. A frequency sweep using sinusoidal excitation was performed to access the number of natural frequencies from 10 to 60 Hz. Whereas only three modes dominated by central tube deflections were found, a large number of lateral stay resonances occur because fabrication imperfections introduce different tension levels in each stay. Many localized modes of vibration occur in the structure as noted by almost random distribution of peaks. The large number of localized modes may be advantageous since energy may be dissipated in local responses rather than global type modes.

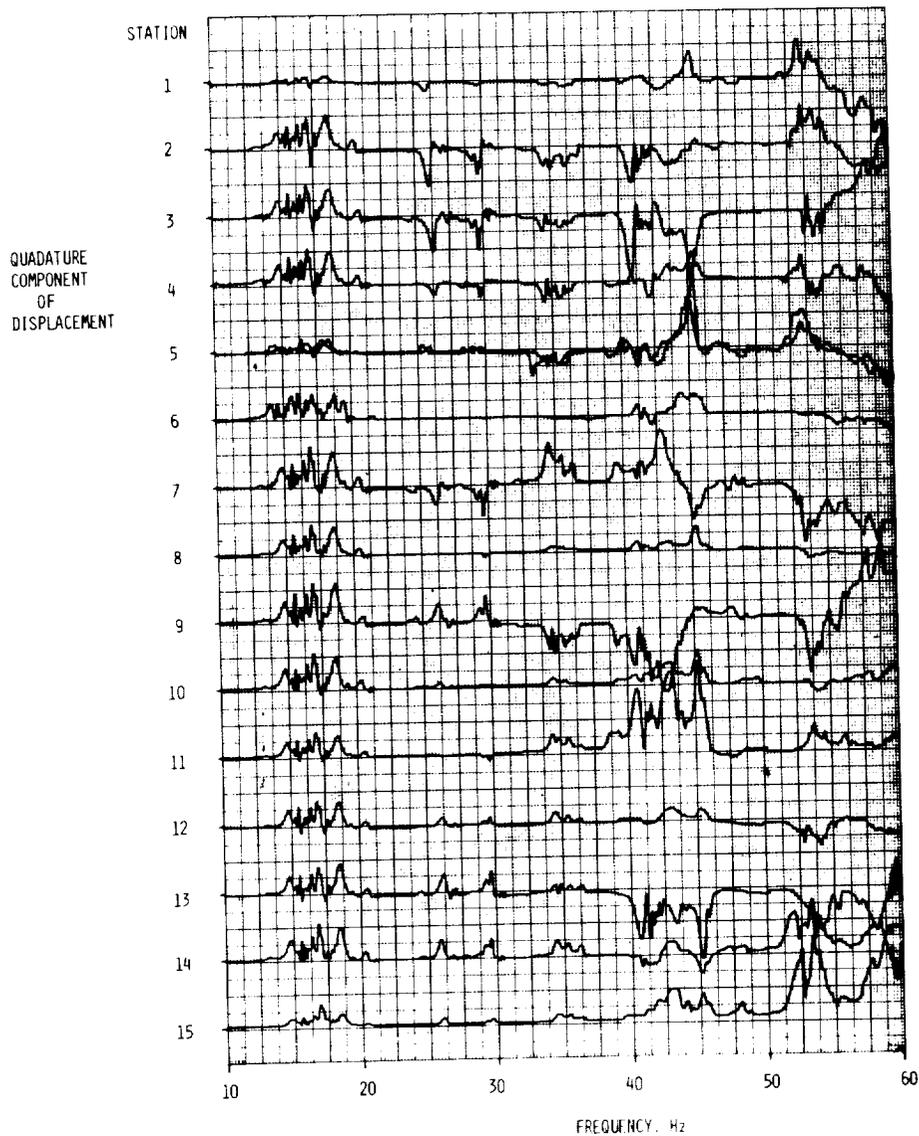


Figure 7

## Fabrication Imperfection Effects on Stay Frequencies

As mentioned earlier, fabrication imperfections can introduce different tension levels in individual stays. The stayed column was designed assuming one stay would be too short by  $\epsilon l_n$  and the two opposite stays too long by  $\epsilon l_n$ . (Where  $\epsilon$  is a measure of fabrication tolerances,  $1.16 \times 10^{-4} \text{m/m}$  for this design, and  $l_n$  is the stay length of the  $n^{\text{th}}$  set.) These imperfections resulted in the design tension levels shown in Fig. 8. Calculated stay frequencies based on the design tension level and measured stay frequencies for the first lateral mode of vibration are also shown in Fig. 8. These data show the deviation in frequency of individual stays created by fabrication imperfections. Since imperfections occur randomly, and the imperfections were not measured, the actual fabrication tolerance ( $\epsilon$ ) has not been determined.

STAY FORCE, N	
N=1	2.4
N=2	1.9
N=3	5.2
N=4	10.6

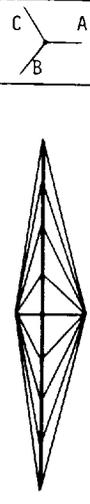
	STAY PLANE	MEASURED B FREQUENCY, Hz			CALCULATED A B C
	A	A	B	C	
	TOP HALF				
	N=4	17.7	14.6	15.6	17.25
	N=3	13.9	17.2	17.0	15.68
	N=2	19.8	18.1	15.4	13.46
	N=1	29.8	22.8	31.2	23.91
	BOTTOM HALF				
	N=1	25.6	22.3	31.1	23.91
	N=2	15.1	15.0	14.7	13.46
	N=3	15.9	16.9	16.1	15.68
	N=4	16.7	16.4	15.2	17.25

Figure 8

## Verification of Dynamic Models

Verification of the stayed column dynamic models was performed by measuring the frequency and shapes of the central tube vibration modes. Two dynamic models were used to predict the central tube modes: Model 1 used linear finite elements with single element representation of the stays and model 2 used a transcendental frequency determinant which accounts for the lateral inertia of the stays. Correlation between analysis and test data of the first three modes is shown in Figure 9. The experimental data is difficult to analyze because of modal coupling between stay and central tube resonances. Nevertheless, both analyses predict the central tube modes with reasonable accuracy. Model 1 and Model 2 predict approximately the same frequency since the stays only represent 8 percent of the total column mass. When the stay mass is not small compared to the effective member mass it is connected to, lateral cable inertia should not be neglected. This inertia can reduce the effective stiffness of the structure and lead to unconservative designs if neglected.

	MODE 1	MODE 2	MODE 3
	FREQUENCY, Hz		
MODEL 1	19.45	36.71	54.64
MODEL 2	19.63	36.09	54.21
EXPERIMENTAL	19.9	34.1	56.5

MODE SHAPES

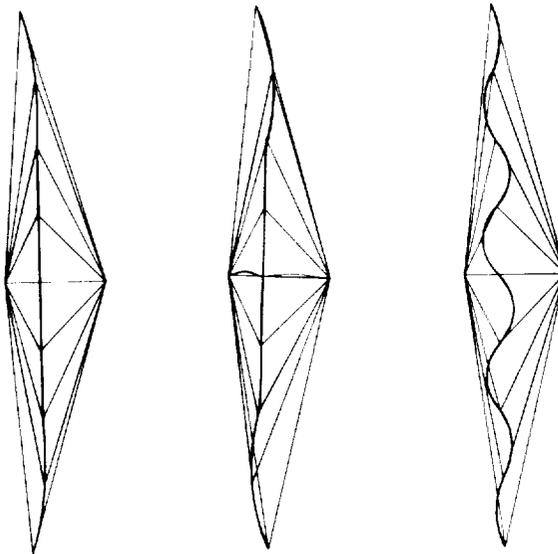


Figure 9

## Nonlinear Vibration-Load Interaction

An important characteristic of any structure is the behavior the structure exhibits under load. The previous figure showed the vibration modes of the central tube with no load. Figure 10 shows the change in frequency of these modes that occurs when a static compressive load is applied to the column. All three modes are reduced in frequency with increasing compressive load. However, the third mode, which has the same mode shape as the buckling mode, is most affected by the compressive load. The data points of Figure 10 were obtained experimentally in order to verify the analysis. At loads below 50 percent of the buckling load the data agree with analysis reasonably well. Above 50 percent of the buckling load nonlinear oscillations occurred. This nonlinear behavior is attributed to isolated stay slackening resulting from beam-column interaction. In a dynamic environment, vibration-load interaction may produce premature collapse of the column due to stay slackening. Consequently, the dynamic environment should be considered in designing the prestress level.

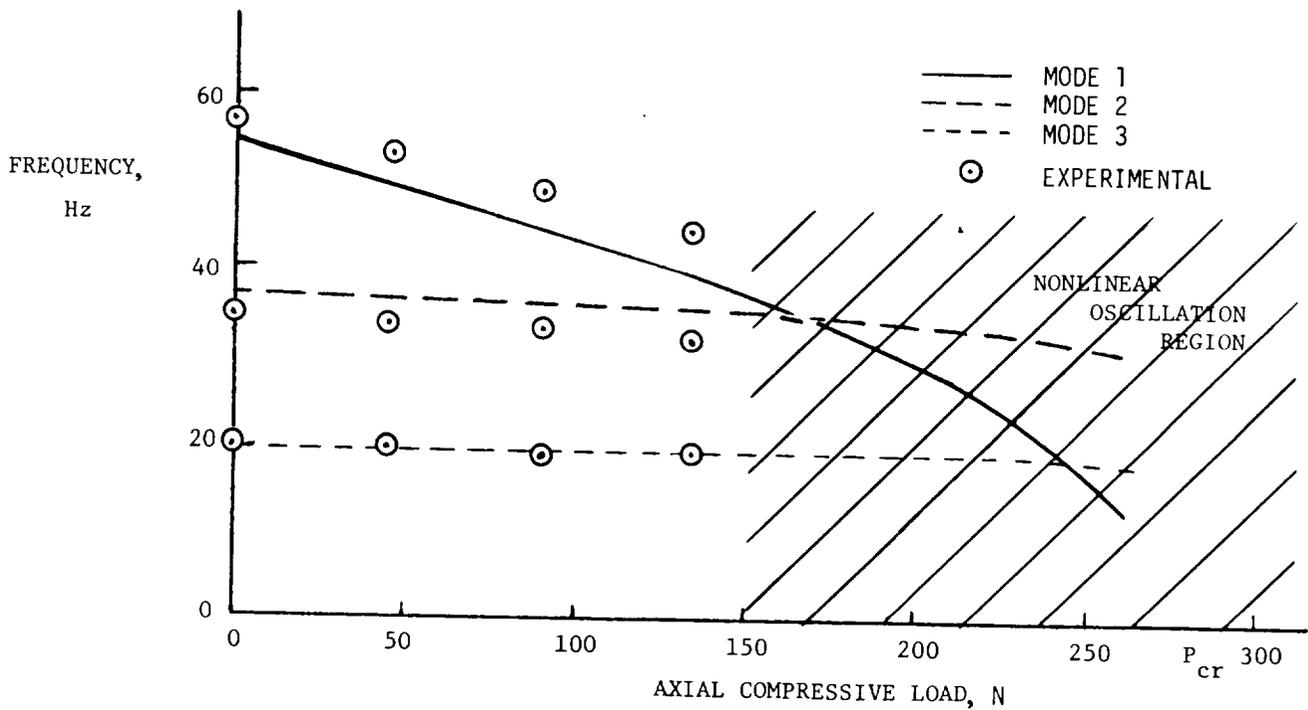


Figure 10

## Effects of Prestress

To prevent stay slackening, the stays are pretensioned during deployment. These pretension forces create precompression in the central tube and spokes. The effect of prestress on the buckling and vibration characteristics of the column was studied by analyzing the column at various prestress levels. Figure 11 shows the change in buckling load with stay pretension and also the change in frequency of the first three central tube modes. Increasing pretension increases precompression in the central tube and thus lowers the buckling load proportionately. A reduction in the frequency of the central tube vibration modes also occurs with increasing pretension. The reduced buckling load and reduced frequencies of vibration are usually considered undesirable characteristics. Consequently, prestress levels are designed to minimize these effects. However, designs based on minimizing stay tension are less conservative since stay slackening is more likely to occur.

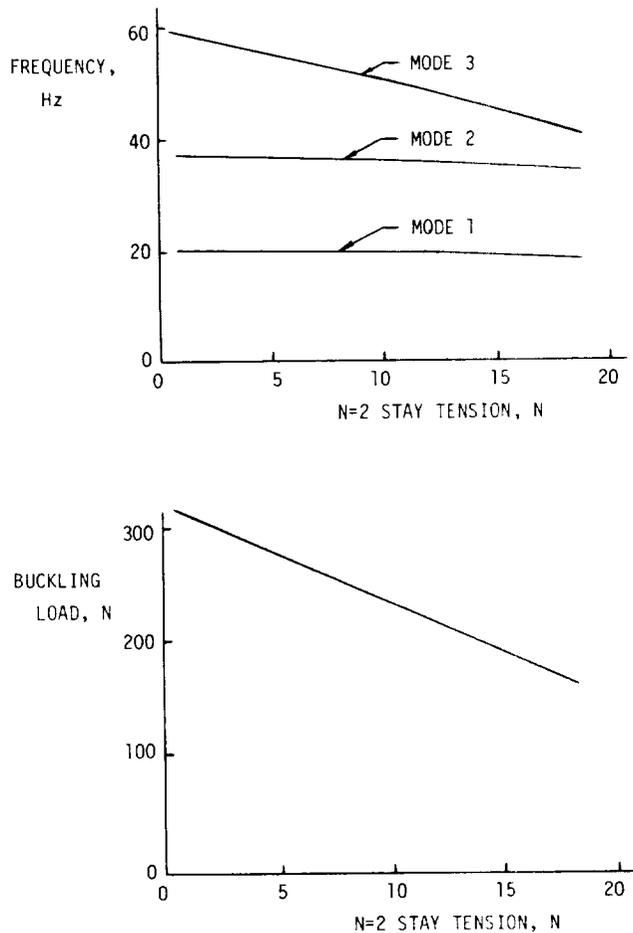


Figure 11

## Summary

Results of analyses and tests of a simple pretensioned structure have been presented. Linear finite element analysis correlated well with experimental small amplitude vibration data. The bifurcation buckling load was also predicted accurately. Postbuckling behavior of the column was unusual and results in a post buckling restoring force of only 1/64 the bifurcation buckling load. Interaction between lateral accelerations and compressive load creates isolated stay slackening at loads above 50 percent of the buckling load. Figure 12 shows several generalized conclusions regarding pretension structures. More research will be required to fully understand their impact on the use of pretensioned structures as large space structures.

- 0 EXPERIMENTAL VERIFICATION OF ANALYTICAL MODELS IS DIFFICULT
- 0 IMPERFECTIONS CREATE UNKNOWN TENSIONS IN INDIVIDUAL STAYS
- 0 DYNAMIC LOADING MUST BE CONSIDERED IN SIZING PRESTRESS LEVELS
- 0 STAY SLACKENING MAY PRODUCE SIGNIFICANT NONLINEARITIES

Figure 12

